Schwartz QFT Solutions

Chapter 2 – Problem 2.1

Start from the Galilean transformation

$$x' = x + vt, \qquad t' = t + \delta t,$$

with $\delta t = \mathcal{O}(v)$ and linear in x, t.

First order in v: Impose invariance of $t^2 - x^2$ to $\mathcal{O}(v)$:

$$(t + \delta t)^2 - (x + vt)^2 = t^2 - x^2 + 2t\,\delta t - 2vxt.$$

Thus,

$$2t\,\delta t - 2vxt = 0 \;\Rightarrow\; \delta t = vx.$$

Hence,

$$x' = x + vt, \qquad t' = t + vx + \mathcal{O}(v^2).$$

Second order in v: Include corrections

$$x' = x + vt + \delta x, \qquad t' = t + vx + \delta t,$$

with

$$\delta x = Av^2 x + Bv^2 t, \qquad \delta t = Cv^2 x + Dv^2 t.$$

Imposing invariance to $\mathcal{O}(v^2)$ gives

$$2t\,\delta t - 2x\,\delta x + v^2(x^2 - t^2) = 0.$$

Matching coefficients:

$$A = \tfrac{1}{2}, \qquad D = \tfrac{1}{2}, \qquad C = B.$$

Choosing $B = 0$,

$$\delta x = \tfrac{1}{2}v^2 x, \qquad \delta t = \tfrac{1}{2}v^2 t.$$

This finally gives,

$$x' = x + vt + \tfrac{1}{2}v^2 x + \mathcal{O}(v^3), \qquad t' = t + vx + \tfrac{1}{2}v^2 t + \mathcal{O}(v^3),$$

which matches the expansion of

$$x' = \frac{x + vt}{\sqrt{1 - v^2}}, \qquad t' = \frac{t + vx}{\sqrt{1 - v^2}} \quad \text{to } \mathcal{O}(v^2).$$