Schwartz QFT Solutions

Chapter 2 – Problem 2.4

Write $x^\mu = (t, x, y, z)$. The transformation is represented by

$$\Lambda = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Since

$$\Lambda^T \eta \Lambda = \eta, \qquad \eta = \text{diag}(1, -1, -1, -1),$$

it preserves the Minkowski metric, so Λ is indeed a Lorentz transformation.

It is not usually listed separately alongside P and T because it is not an independent discrete generator. In fact,

$$P : (t, x, y, z) \mapsto (t, -x, -y, -z),$$

$$R_y(\pi) : (t, x, y, z) \mapsto (t, -x, y, -z).$$

Therefore

$$P\, R_y(\pi) : (t, x, y, z) \mapsto (t, x, -y, z) = \Lambda.$$

So Λ is just parity followed by an ordinary proper rotation.