Schwartz QFT Solutions

Chapter 2 – Problem 2.5

Let the incoming photon four-momentum be

$$k^\mu = (\omega, \mathbf{k}), \qquad |\mathbf{k}| = \omega,$$

and the outgoing photon

$$k'^\mu = (\omega', \mathbf{k}'), \qquad |\mathbf{k}'| = \omega'.$$

Let the electron momenta be

$$p^\mu = (m, \mathbf{0}), \qquad p'^\mu = (E', \mathbf{p}').$$

Define the scattering angle by

$$\cos\theta = \frac{\mathbf{k}\cdot\mathbf{k}'}{\omega\omega'}.$$

(a)

The X-ray energy is much larger than the electron binding energy in the crystal, so the electron behaves as a free particle during the scattering. Hence we may impose free-particle energy-momentum conservation and on-shell conditions.

(b)

Energy-momentum conservation:

$$p + k = p' + k'.$$

Imposing the on-shell condition $p'^2 = m^2$,

$$(p + k - k')^2 = m^2.$$

Expanding,

$$p^2 + k^2 + k'^2 + 2p\cdot k - 2p\cdot k' - 2k\cdot k' = m^2.$$

Using

$$p^2 = m^2, \quad k^2 = 0, \quad k'^2 = 0,$$

we get

$$p\cdot k - p\cdot k' = k\cdot k'.$$

$$m\omega = \omega'\big(m + \omega(1-\cos\theta)\big),$$

$$\omega'(\theta) = \frac{\omega}{1 + \frac{\omega}{m}(1-\cos\theta)}$$

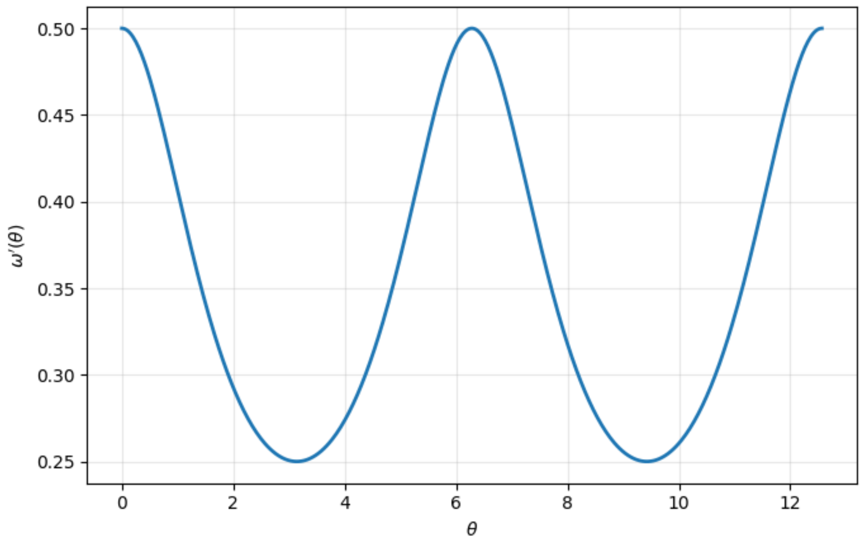

Figure 1: Scattered photon frequency as a function of angle.

(c)

For $\theta \neq 0$,

$$\omega'(\theta) = \frac{\omega}{1 + \frac{\omega}{m}(1 - \cos\theta)} \longrightarrow 0 \quad (m \to 0).$$

While

$$\omega'(0) = \omega.$$

Thus the distribution collapses to forward scattering.

$$\omega'(\theta) \to \begin{cases} \omega, & \theta = 0 \\ 0, & \theta \neq 0 \end{cases} \quad (m \to 0)$$

(d)

Classically, the electron re-radiates at the same frequency as the incident wave, so no frequency shift is expected:

$$\omega' = \omega.$$

Thus the spectrum would be

$$\frac{dI}{d\omega'} \propto \delta(\omega' - \omega).$$